Table 8.1. The Michell solution — stress components

ϕ	σ_{rr}	$\sigma_{r\theta}$	$\sigma_{\theta\theta}$
r^2	2	0	2
$r^2\ln(r)$	$2\ln(r)+1$	0	$2\ln(r)+3$
$\ln(r)$	$1/r^2$	0	$-1/r^2$
θ	0	$1/r^2$	0
$r^3\cos\theta$	$2r\cos\theta$	$2r\sin\theta$	$6r\cos\theta$
$r\theta\sin\theta$	$2\cos\theta/r$	0	0
$r\ln(r)\cos\theta$	$\cos\theta/r$	$\sin\theta/r$	$\cos\theta/r$
$\cos\theta/r$	$-2\cos\theta/r^3$	$-2\sin\theta/r^3$	$2\cos\theta/r^3$
$r^3\sin\theta$	$2r\sin\theta$	$-2r\cos\theta$	$6r\sin\theta$
$r\theta\cos\theta$	$-2\sin\theta/r$	0	0
$r\ln(r)\sin\theta$	$\sin\theta/r$	$-\cos\theta/r$	$\sin\theta/r$
$\sin\theta/r$	$-2\sin\theta/r^3$	$2\cos\theta/r^3$	$2\sin\theta/r^3$
$r^{n+2}\cos n\theta$	$-(n+1)(n-2)r^n\cos n\theta$	$n(n+1)r^n\sin n\theta$	$(n+1)(n+2)r^n\cos n\theta$
$r^{-n+2}\cos n\theta$	$-(n+2)(n-1)r^{-n}\cos n\theta$	$-n(n-1)r^{-n}\sin n\theta$	$(n-1)(n-2)r^{-n}\cos n\theta$
$r^n\cos n\theta$	$-n(n-1)r^{n-2}\cos n\theta$	$n(n-1)r^{n-2}\sin n\theta$	$n(n-1)r^{n-2}\cos n\theta$
$r^{-n}\cos n\theta$	$-n(n+1)r^{-n-2}\cos n\theta$	$-n(n+1)r^{-n-2}\sin n\theta$	$n(n+1)r^{-n-2}\cos n\theta$
$r^{n+2}\sin n\theta$	$-(n+1)(n-2)r^n\sin n\theta$	$-n(n+1)r^n\cos n\theta$	$(n+1)(n+2)r^n\sin n\theta$
$r^{-n+2}\sin n\theta$	$-(n+2)(n-1)r^{-n}\sin n\theta$	$n(n-1)r^{-n}\cos n\theta$	$(n-1)(n-2)r^{-n}\sin n\theta$
$r^n\sin n\theta$	$-n(n-1)r^{n-2}\sin n\theta$	$-n(n-1)r^{n-2}\cos n\theta$	$n(n-1)r^{n-2}\sin n\theta$
$r^{-n}\sin n\theta$	$-n(n+1)r^{-n-2}\sin n\theta$	$n(n+1)r^{-n-2}\cos n\theta$	$n(n+1)r^{-n-2}\sin n\theta$

8.4.1 Hole in a tensile field

To illustrate the use of Table 8.1, we consider the case where the the body of Figure 8.2 is subjected to uniform tension at infinity instead of shear, so that the boundary conditions become

$$\sigma_{rr} = 0 \;;\; r=a \tag{8.60}$$

$$\sigma_{r\theta} = 0 \;;\; r=a \tag{8.61}$$

$$\sigma_{xy}, \sigma_{yy} \to 0 \;;\; r\to\infty \tag{8.62}$$

$$\sigma_{xx} \to S \;;\; r\to\infty\,. \tag{8.63}$$

The unperturbed problem in this case can clearly be described by the stress function

$$\phi = \frac{Sy^2}{2} = \frac{Sr^2\sin^2\theta}{2} = \frac{Sr^2}{4} - \frac{Sr^2\cos(2\theta)}{4}\,. \tag{8.64}$$

Table 9.1. The Michell solution — displacement components

ϕ	$2\mu u_r$	$2\mu u_\theta$
r^2	$(\kappa-1)r$	0
$r^2\ln(r)$	$(\kappa-1)r\ln(r)-r$	$(\kappa+1)r\theta$
$\ln(r)$	$-1/r$	0
θ	0	$-1/r$
$r^3\cos\theta$	$(\kappa-2)r^2\cos\theta$	$(\kappa+2)r^2\sin\theta$
$r\theta\sin\theta$	$\frac{1}{2}\{(\kappa-1)\theta\sin\theta-\cos\theta$ $+(\kappa+1)\ln(r)\cos\theta\}$	$\frac{1}{2}\{(\kappa-1)\theta\cos\theta-\sin\theta$ $-(\kappa+1)\ln(r)\sin\theta\}$
$r\ln(r)\cos\theta$	$\frac{1}{2}\{(\kappa+1)\theta\sin\theta-\cos\theta$ $+(\kappa-1)\ln(r)\cos\theta\}$	$\frac{1}{2}\{(\kappa+1)\theta\cos\theta-\sin\theta$ $-(\kappa-1)\ln(r)\sin\theta\}$
$\cos\theta/r$	$\cos\theta/r^2$	$\sin\theta/r^2$
$r^3\sin\theta$	$(\kappa-2)r^2\sin\theta$	$-(\kappa+2)r^2\cos\theta$
$r\theta\cos\theta$	$\frac{1}{2}\{(\kappa-1)\theta\cos\theta+\sin\theta$ $-(\kappa+1)\ln(r)\sin\theta\}$	$\frac{1}{2}\{-(\kappa-1)\theta\sin\theta-\cos\theta$ $-(\kappa+1)\ln(r)\cos\theta\}$
$r\ln(r)\sin\theta$	$\frac{1}{2}\{-(\kappa+1)\theta\cos\theta-\sin\theta$ $+(\kappa-1)\ln(r)\sin\theta\}$	$\frac{1}{2}\{(\kappa+1)\theta\sin\theta+\cos\theta$ $+(\kappa-1)\ln(r)\cos\theta\}$
$\sin\theta/r$	$\sin\theta/r^2$	$-\cos\theta/r^2$
$r^{n+2}\cos n\theta$	$(\kappa-n-1)r^{n+1}\cos n\theta$	$(\kappa+n+1)r^{n+1}\sin n\theta$
$r^{-n+2}\cos n\theta$	$(\kappa+n-1)r^{-n+1}\cos n\theta$	$-(\kappa-n+1)r^{-n+1}\sin n\theta$
$r^n\cos n\theta$	$-nr^{n-1}\cos n\theta$	$nr^{n-1}\sin n\theta$
$r^{-n}\cos n\theta$	$nr^{-n-1}\cos n\theta$	$nr^{-n-1}\sin n\theta$
$r^{n+2}\sin n\theta$	$(\kappa-n-1)r^{n+1}\sin n\theta$	$-(\kappa+n+1)r^{n+1}\cos n\theta$
$r^{-n+2}\sin n\theta$	$(\kappa+n-1)r^{-n+1}\sin n\theta$	$(\kappa-n+1)r^{-n+1}\cos n\theta$
$r^n\sin n\theta$	$-nr^{n-1}\sin n\theta$	$-nr^{n-1}\cos n\theta$
$r^{-n}\sin n\theta$	$nr^{-n-1}\sin n\theta$	$-nr^{-n-1}\cos n\theta$

condition that the body be in equilibrium. This requires that

$$\int_0^{2\pi}(F_1(\theta)\cos\theta-F_3(\theta)\sin\theta)\,ad\theta - \int_0^{2\pi}(F_2(\theta)\cos\theta-F_4(\theta)\sin\theta)bd\theta=0 \tag{9.36}$$

$$\int_0^{2\pi}(F_1(\theta)\sin\theta+F_3(\theta)\cos\theta)ad\theta - \int_0^{2\pi}(F_2(\theta)\sin\theta+F_4(\theta)\cos\theta)bd\theta=0 \tag{9.37}$$