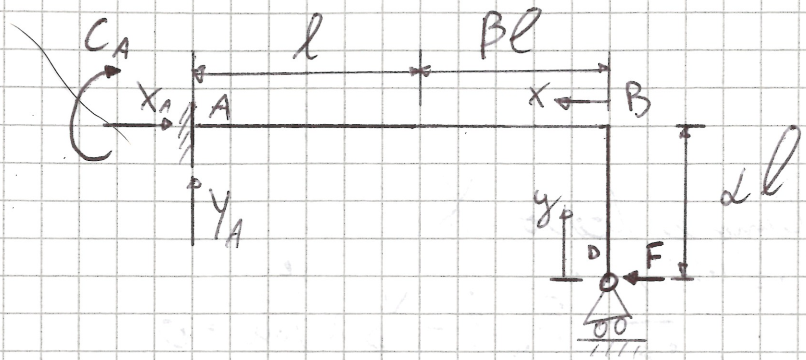
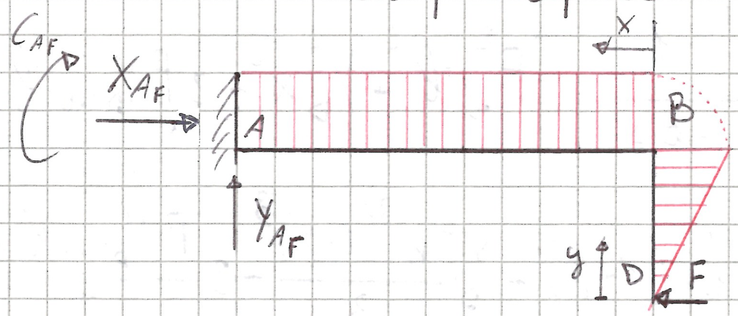
Esercizio 2.06



Considero la struttura principale caricata dalla sola forza F.



$\uparrow]\ Y_{A_F} = 0$

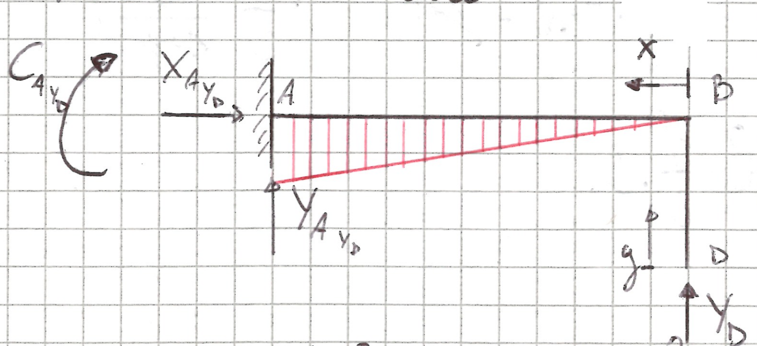
$\rightarrow]\ X_{A_F} = F$

$\circlearrowleft_A]\ C_{A_F} = -F \cdot \alpha \cdot \ell$

$Mf_{BA_F}(x) = F \cdot \alpha \ell$

$Mf_{DB_F}(y) = F \cdot y$

Considero la struttura



$\uparrow]\ Y_{A_{Y_D}} = -Y_D$ $\circlearrowleft]\ C_{A_{Y_D}} = Y_D \cdot (\ell + \beta\ell) = Y_D \cdot \ell(1+\beta)$

$\rightarrow]\ X_{A_{Y_D}} = 0$

$$M_{f_{BA_{Y_D}}}(x) = -Y_D \cdot x$$

$$M_{f_{DB_{Y_D}}}(y) = 0$$

Applico il teorema di Castigliano e trovo Y_D.

$$U = \int_0^{\alpha l} \frac{1}{2EJ}\left(F \cdot y + 0\right)^2 dy + \int_0^{l(1+\beta)} \frac{1}{2EJ}\left(F \cdot \alpha \cdot l - Y_D \cdot x\right)^2 dx =$$

$$= \int_0^{\alpha l} \frac{1}{2EJ} F^2 \cdot y^2 \, dy + \int_0^{l(1+\beta)} \frac{1}{2EJ}\left(F^2 \alpha^2 l^2 + Y_D^2 x^2 - 2 F \cdot Y_D \cdot \alpha \cdot l \cdot x\right) dx =$$

$$= \frac{1}{2EJ}\left(F^2 \cdot \frac{\alpha^3 l^3}{3} + F^2 \cdot \alpha^2 \cdot l^2 \cdot l \cdot (1+\beta) + Y_D^2 \cdot \frac{l^3 (1+\beta)^3}{3} - \cancel{2} \frac{F \cdot Y_D \cdot \alpha \cdot l \cdot l^2 (1+\beta)^2}{\cancel{2}}\right)$$

$$f_D = 0 = \frac{\partial U}{\partial Y_D} = 0 + 0 + 2 Y_D \cdot \frac{l^3 (1+\beta)^3}{3} - F \cdot \alpha \cdot l^3 (1+\beta)^2$$

$$\Rightarrow Y_D = F \cdot \alpha \cdot \cancel{l^3 (1+\beta)^2} \cdot \frac{3}{2 \cancel{l^3} (1+\beta)^{\cancel{3}}} = \frac{F \cdot 3 \cdot \alpha}{2(1+\beta)}$$

$$X_A = 0 + F = F$$

$$Y_A = 0 - Y_D = -F \cdot \frac{3}{2} \cdot \frac{\alpha}{1+\beta}$$

$$C_A = -F \cdot \alpha \cdot l + Y_D \cdot l \cdot (1+\beta) = l\left(-F \cdot \alpha + \cancel{(1+\beta)} \cdot F \cdot \frac{3}{2} \cdot \frac{\alpha}{\cancel{1+\beta}}\right)$$

$$= F \cdot l \left(-\alpha + \frac{3}{2}\alpha\right) = F \cdot l \cdot \frac{1}{2} \alpha$$